# Possibilistic vs Evidential Valuation Algebra Networks

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Department of Defence
Science and Technology

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# Outline

1. Realistic Reasoning Applications
   - Heterogeneous data, various types of uncertainty, multiple variables
   - Illustrative example $\longrightarrow$ The Captain's Decision Problem
2. Approaches to Modelling Uncertainty
   - Handling multiple variables $\longrightarrow$ Valuation Based Algebra (VBA)
   - Beyond probability theory $\longrightarrow$ Possibility theory, DS evidence theory
3. New Possibilistic Valuation Algebra Network
   - Valuations as Possibility Functions
   - VBA Operations adhere to Possibility Theory
   - Possibilistic Uncertain Implication Rule
4. Simulation Results
5. Conclusions

# Real World Reasoning and Decision Making...

Some challenges:

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- Domain knowledge
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- Multivariate
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e.g.

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- System reliability

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Our approach:

- Model multivariate systems using Valuation Algebra Networks
- Construct reasoning networks using different models of uncertainty e.g. Possibility Theory and Dempster-Shafer Evidence Theory

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# An example - The Captain's Decision Problem[1]

Estimate the number of days a ship will be delayed based on the following

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A framework for representing knowledge and inferring outcomes within a system

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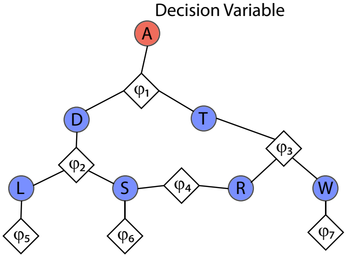
A framework for representing knowledge and inferring outcomes within a system

Primary Elements:

1. Variables $V$ within the system
2. Valuation functions $\varphi$

e.g. Captain's Decision Problem




Valuations represent knowledge about the relationship between variables

Notation: $\Theta_{\mathcal{D}}$ = set of possible values of a set of variables $\mathcal{D}$
$\looparrowright$ = Configurations of $\mathcal{D}$

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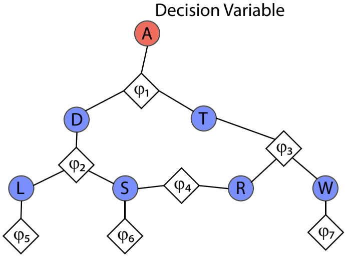


Combination is the aggregation of knowledge

e.g. $\varphi_2 \oplus \varphi_5 =$ aggregated knowledge from $\varphi_2$ and $\varphi_5$

Marginalization is the focusing of knowledge

e.g. $\varphi_2^{\downarrow D} =$ knowledge of $D$ implied by $\varphi_2$ if other variables disregarded.

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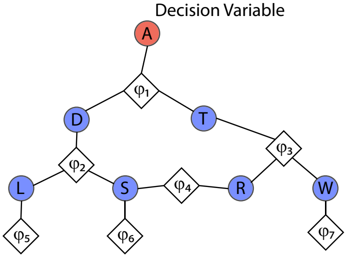


Inference = Successive application of combination & marginalization

↬ e.g. end goal is to obtain $(\varphi_1 \oplus \varphi_2 \oplus \ldots \oplus \varphi_7)^{\downarrow A}$

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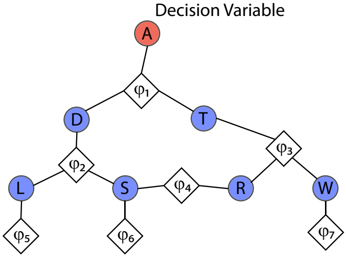


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**If $\{\oplus, \downarrow\}$ satisfy a set of axioms[2] = marginal can be computed locally!**

[2] P. Shenoy and G. Shafer, '*Axioms for probability and belief-function propagation*', in Readings in uncertain reasoning, 1990.

# Quantifying Uncertainty in a System

↬ Specifying the valuation functions and the operations $\oplus, \downarrow$

Many approaches to modeling uncertainty:

- Probability theory
- Possibility theory[1]
- Imprecise probabilities[2]
- Random sets[3]
- Dempster-Shafer (DS) evidence theory[4,5]

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Compare two approaches:

VBA using DS evidence theory $\Longrightarrow$ Evidential Networks[6]
VBA using Possibility theory $\Longrightarrow$ Possibilistic Networks **(our focus)**

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# Recap on Zadeh's Possibility Theory

Principle of minimal specificity

Unless impossible, no hypothesis can be ruled out

## Measuring Possibility

Given a set of variables $\mathcal{D}$ with configurations $\Theta_{\mathcal{D}}$:

$\looparrowright$ possibility of an event $A \subseteq \Theta_{\mathcal{D}}$ is the mapping $\Pi : 2^{\Theta_{\mathcal{D}}} \to [0, 1]$

where $2^{\Theta_{\mathcal{D}}}$ is the power set of $\Theta_{\mathcal{D}}$.

$\Pi$ satisfies:

1. $\Pi(\varnothing) = 0 \to \Theta_{\mathcal{D}}$ is an exhaustive set of configurations
2. $\Pi(\Theta_{\mathcal{D}}) = 1 \to$ mapping $\Pi$ is free of contradictions
3. $\Pi(A_1 \cup A_2) = \max(\Pi(A_1), \Pi(A_2)) \to$ replaces the additivity axiom in probability theory.

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Concept of Necessity $N$:

Necessity is the dual of possibility such that

$N(A) = 1 - \Pi(A^c)$, where $A^c$ is the complement of $A$

# Possibility Functions

## Possibility Functions

A function $\pi : \Theta_{\mathcal{D}} \to [0,1]$ such that, for $A \subseteq \Theta_{\mathcal{D}}$:

$$\Pi(A) = \max_{x \in A} \pi(x) \quad \text{and} \quad N(A) = \min_{x \in A^c} (1 - \pi(x))$$

Properties:

- $\pi(x) = 1$ for $x = x_0$ and 0 otherwise $\implies$ Complete knowledge
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Relationship to a probability function $p(x)$:

$$\pi(x) = \frac{p(x)}{\max_{x \in \Theta_{\mathcal{D}}} p(x)} \iff p(x) = \frac{\pi(x)}{\sum_{x \in \Theta_{\mathcal{D}}} \pi(x)}$$

# Possibilistic Networks

Key Concepts:
- Valuation functions = Possibility functions
- Operations adhere to Possibility Theory

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# Possibilistic Networks

Key Concepts: $\begin{cases} \text{Valuation functions} = \text{Possibility functions} \\ \text{Operations adhere to Possibility Theory} \end{cases}$

## Possibilistic Combination[1]:

Given $\pi_1^{\mathcal{D}}$ and $\pi_2^{\mathcal{D}}$ representing valuations $\varphi_1$ and $\varphi_2$ then:

$$\left(\pi_1^{\mathcal{D}} \oplus \pi_2^{\mathcal{D}}\right)(a) = \frac{\pi_1^{\mathcal{D}}(a) \cdot \pi_2^{\mathcal{D}}(a)}{\max_{a \in \Theta_{\mathcal{D}}} \left\{\pi_1^{\mathcal{D}}(a) \cdot \pi_2^{\mathcal{D}}(a)\right\}}, \quad \forall \quad a \in \Theta_{\mathcal{D}}$$

## Possibilistic Marginalization[1]:

Given $\pi^{\mathcal{D}}$ then marginalization onto a coarser domain $\mathcal{D}' \subseteq \mathcal{D}$ is:

$$\pi^{\downarrow \mathcal{D}'}(a) = \max_{b \in \Theta_{\mathcal{D}}} \left\{\pi^{\mathcal{D}}(a, b)\right\}, \quad \forall \quad a \in \Theta_{\mathcal{D}'}.$$

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↬ Operations satisfy axioms for local computation

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# Possibilistic Uncertain Implication Rules

Given outcomes $a \in \Theta_{\mathcal{D}_1}$ and $b \in \Theta_{\mathcal{D}_2}$:

Rule allows evaluation of expert knowledge of the form "if $a$ then $b$" with an associated confidence level $[\alpha, \beta]$, where $0 \leq \alpha \leq \beta \leq 1$.

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We formulate possibilistic rule based on:

- If $a^c \implies$ Complete ignorance on outcomes in $\Theta_{\mathcal{D}_2}$
- If $a$ and $b \implies$ Confidence in outcome $\max([\alpha, \beta]) = \beta$
- If $a$ and $b^c \implies$ Confidence in outcome $\max([1-\beta, 1-\alpha]) = 1-\alpha$
- Normalise outcomes dependent on $a$

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Thus we propose:

$$\pi_R^{\mathcal{D}_1 \times \mathcal{D}_2}(c) = \begin{cases} \beta K^{-1} & \text{if } c = a \times b \\ (1-\alpha)K^{-1} & \text{if } c = a \times b^c \\ 1 & \text{if } c = a^c \times \Theta_{\mathcal{D}_2} \end{cases}$$

where $K = \max(\beta, 1-\alpha)$ and $\times$ represents the Cartesian product.

# Comparing Networks using the Captain's Problem

Scenario A - using only prior information:

## Prior Information

1. Arrival delay (A) is due to departure delay (D) and the travel delay (T).
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Scenario B - using prior and additional information:

## Additional Information

5. Chance of 1 day loading delay is 30% to 50%.
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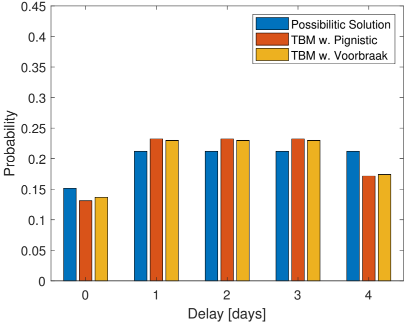
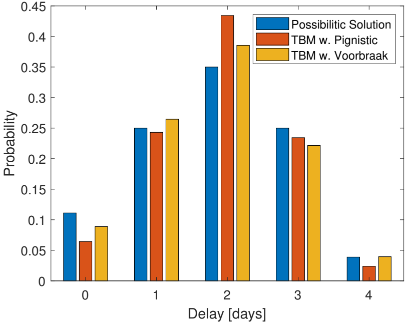
# Results - Scenarios A & B

Comparing possibilistic and evidential networks:




Scenario (A)




Scenario (B)

$\looparrowright$ Prior information fairly uninformative on its own

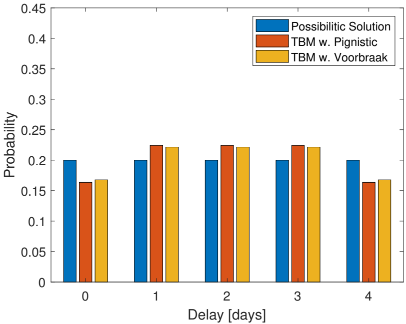
Using additional information $\Longrightarrow$ All networks predict 2 days delay

TBM w. Pignistic = evidential network using Pignistic transform

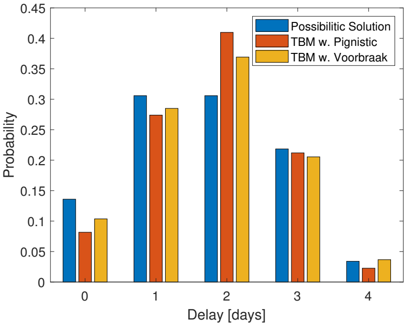
TBM w. Voorbraak = evidential network using Voorbraak transform

# Results - Probing Network Performance

Change implication rule in the prior information:




Scenario (C) - New Prior Information




Scenario (D) - New Prior + Additional

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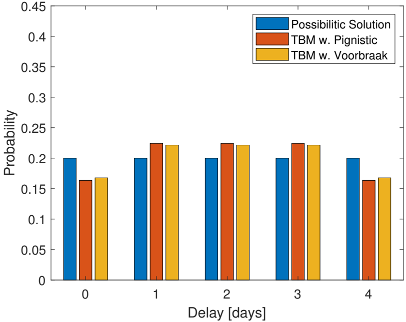


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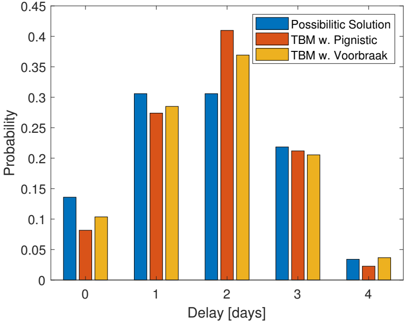


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↬ Change in prediction for the possibilistic network

# Further Analysis...

Determine the probability of requiring a repair at set by analysing the local network relating $S$ and $R$:

| VBS Network | Scenario (B) | | Scenario (D) | |
|---|---|---|---|---|
| | Repair | No Repair | Repair | No Repair |
| Possibilistic | 0.583 | 0.417 | 0.500 | 0.500 |
| Evidential with Pignistic | 0.750 | 0.250 | 0.650 | 0.350 |
| Evidential with Voorbraak | 0.667 | 0.333 | 0.588 | 0.412 |

↬ Difference in behaviour due to uncertain implication rule

# Conclusions & Future Work

## Conclusions

- Focus on reasoning in uncertain multivariate systems
  - Model system $\Longrightarrow$ Valuation base algebra framework
  - Model uncertainty $\Longrightarrow$ Possibility theory or DS evidence theory
- Presented possibilistic VBA network
  - Valuations and operations adhere to Possibility theory
  - Proposed possibilistic uncertain implication rule
- Compared evidential & possibilistic networks on Captain's Problem
  - Possibilistic network sensitive to changes in implication rule

## Future Work

Open problem:

$\looparrowright$ Develop a framework for evaluating VBS networks

## The End

# Thank you for listening