Local All-Pass Filters for Optical Flow Estimation

Christopher Gilliam and Thierry Blu

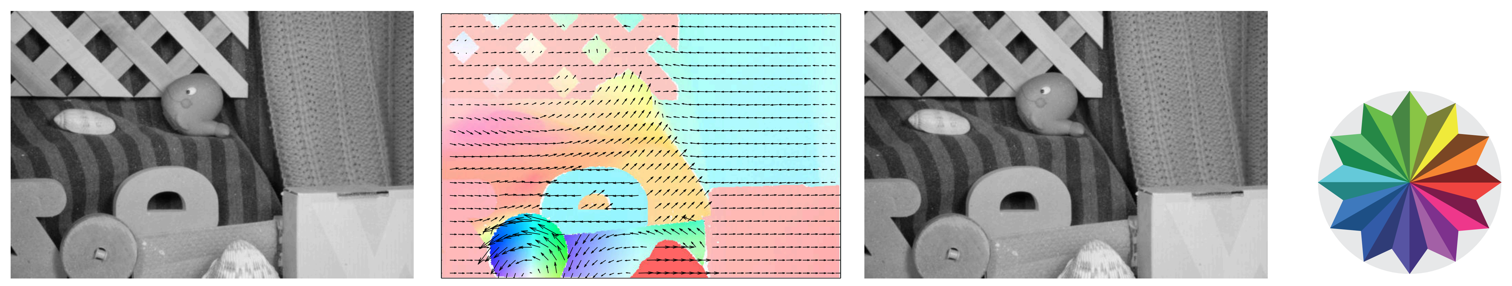
Department of Electronic Engineering, The Chinese University of Hong Kong email: {cgilliam, tblu}@ee.cuhk.edu.hk

Summary

An important topic in image processing is the estimation of motion from a sequence of images. This motion is known as the **Optical Flow** and is utilised in a range of applications e.g. computer vision, biology and medical imaging. In this work, we present a novel algorithm to estimate the optical flow using local all-pass filters. We demonstrate that this algorithm is fast, consistent, and that it outperforms three state-of-the-art algorithms when estimating constant and smoothly varying flows. We also show initial competitive results for real images.

Optical Flow Estimation

Problem: Find a velocity field $\mathrm{u}(x,y) = [u_1(x,y), u_2(x,y)]^{\mathrm{T}}$ based on the variation of pixel intensities within an image sequence [1], where (x,y) is the pixel coordinates.



(a) Image 1, $I_1(x,y)$ (b) Optical Flow, $\mathrm{u}(x,y)$ (c) Image 2, $I_2(x,y)$ (d) Flow Colour Code

Standard Framework

Assume a pixel's intensity remains constants as it flows from one image to another:

$$\text{Brightness Constraint:} \quad \underbrace{I_2(x,y) = I_1(x - u_1(x,y), y - u_2(x,y))}_{\text{Non-Linear}}$$

Linearise constraint by performing first order Taylor approximation under the assumption that the displacement of the optical flow is small [1,2]:

$$\text{Optical Flow Equation:} \quad \underbrace{I_2 - I_1 + u_1\frac{\partial I_1}{\partial x} + u_2\frac{\partial I_2}{\partial y} = 0}_{\text{1 Constraint for 2 Unknowns} \Rightarrow \text{Ill-posed (Aperture Problem)}}$$

Overcoming the Aperture Problem:

Global Approach: Minimise a global energy function that comprises the optical flow equation as a data term and a regularisation constraint on the flow as a prior term [1].

Local Approach: Constrain the optical flow to be constant over a local region and solve the optical flow equation within the region [2].

Our Approach

Instead of assuming small displacement and using the optical flow equation:

$$\boxed{\text{Assume the optical flow is slowly varying} \Rightarrow \text{Treat as locally constant}}$$

Under this assumption:

- Relate local changes between two images via a filter that is **All-Pass** in nature
- Extract local estimate of optical flow from this all-pass filter

↬ No limit on the size of displacement of the flow

All-Pass Filtering Framework

1. Shifting is All-Pass Filtering

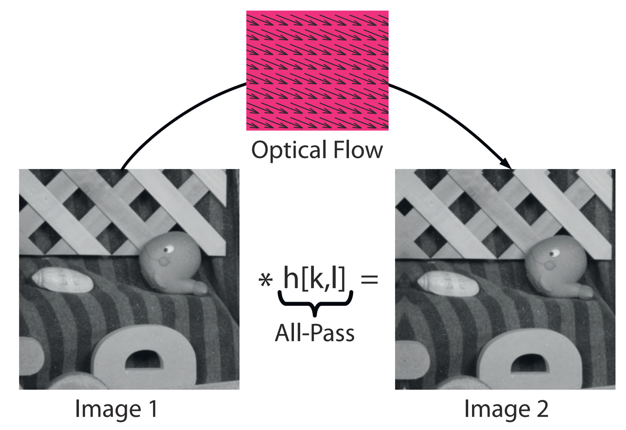
Under brightness constraint:

Constant optical flow $\Longrightarrow$ Shifting by a displacement vector $\mathrm{u} = [u_1, u_2]^{\mathrm{T}}$

Shifting in frequency domain:

$$\hat{I}_2(\omega_1,\omega_2) = \underbrace{\hat{I}_1(\omega_1,\omega_2)\,\mathrm{e}^{-ju_1\omega_1 - ju_2\omega_2}}_{=\ \text{Filtering Operation}}$$

$$\text{All-Pass Filter:} \quad H(\omega_1,\omega_2) = \mathrm{e}^{-ju_1\omega_1 - ju_2\omega_2}$$



2. Rational Representation of All-Pass Filter

The $(2\pi, 2\pi)$-periodic frequency response of any digital all-pass filter can be expressed as:

$$H(\omega_1,\omega_2) = \frac{P\left(\mathrm{e}^{j\omega_1}, \mathrm{e}^{j\omega_2}\right)}{P\left(\mathrm{e}^{-j\omega_1}, \mathrm{e}^{-j\omega_2}\right)} \quad \begin{array}{l} \longleftrightarrow \quad \text{Forward Filter} \\ \longleftrightarrow \quad \text{Backward Filter} \end{array}$$

Linearise filtering performed by h:

$$I_2[k,l] = h[k,l] * I_1[k,l] \quad \Longleftrightarrow \quad p[-k,-l] * I_2[k,l] = p[k,l] * I_1[k,l]$$

3. Filter Approximation - A Basis Representation

Approximate p using a linear combination of a few, known, real filters:

$$p_{\text{app}}[k,l] = \sum_{n=0}^{N-1} c_n p_n[k,l]$$

Opt for compact filter basis based on Gaussian filters:

$$\begin{aligned} p_0[k,l] &= \mathrm{e}^{-\frac{k^2+l^2}{2\sigma^2}} & p_3[k,l] &= (k^2+l^2-2\sigma^2)p_0[k,l] \\ p_1[k,l] &= k\,p_0[k,l] & p_4[k,l] &= kl\,p_0[k,l] \\ p_2[k,l] &= l\,p_0[k,l] & p_5[k,l] &= (k^2-l^2)\,p_0[k,l] \end{aligned}$$

where $\sigma = (R+2)/4$ and R is the half-support of the filters.

4. Extracting the Displacement Vector

$$\text{Since } H_{\text{app}} \approx \mathrm{e}^{-ju_1\omega_1 - ju_2\omega_2} \quad \Longrightarrow \quad u_{1,2} = j\left.\frac{\partial \log\left(H_{\text{app}}\left(\mathrm{e}^{j\omega_1}, \mathrm{e}^{j\omega_2}\right)\right)}{\partial \omega_{1,2}}\right|_{\omega_1=\omega_2=0}$$

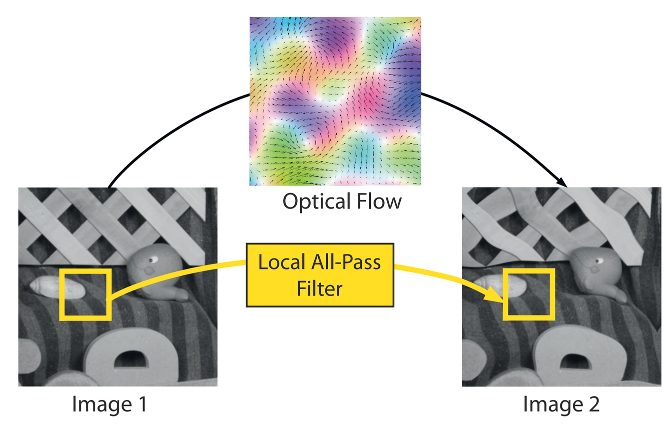
Local All-Pass Algorithm

Assume flow is constant within a window $\mathcal{R}$ and estimate a local all-pass filter. Thus, for $(2R+1)$ square window $\mathcal{R}$, solve at every pixel:

$$\min_{\{c_n\}} \sum_{k,l\in\mathcal{R}} \left|p_{\text{app}}[-k,-l] * I_2[k,l] - p_{\text{app}}[k,l] * I_1[k,l]\right|^2$$

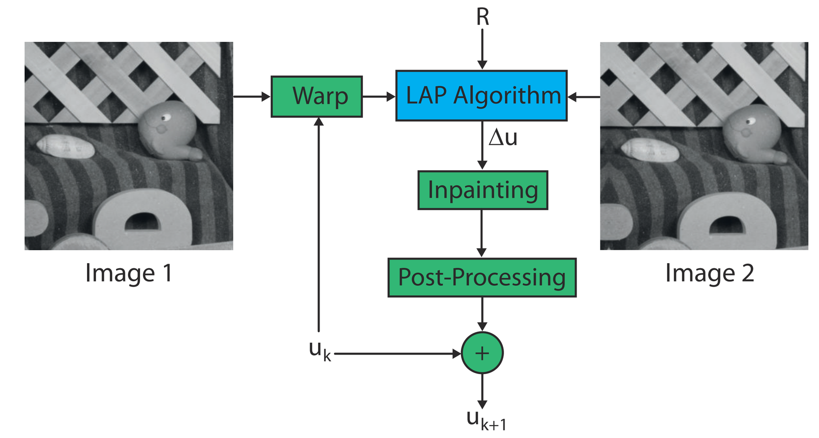
↬ $c_0 = 1 \implies$ Solve linear system of equations with $N-1$ unknowns

- Efficient implementation using convolutions and pointwise multiplication
- Extract optical flow estimate from filters



Multi-Scale Refinement

Estimate the flow in a slow-to-fast varying manner by changing the filter parameter R; large values of R allow the estimation of large flow whilst small values allow faster variations.



Post-Processing:

- Remove erroneous flow estimates using inpainting
- Smooth flow estimate using mean filtering

↬ Real Images $\Longrightarrow$ Pre-process images using high-pass filter and median filtering at small R

Results

Evaluation under two conditions:

Noiseless Conditions: Image I_2 is generated by directly warping image I_1 with a synthetic optical flow. Therefore, the images exactly satisfy brightness constraint.

Real Conditions: Image I_2 is acquired independently of I_1. Therefore, the images are unlikely to satisfy the brightness constraint exactly (i.e. noisy conditions).

Accuracy:

$$\text{Measures:} \quad \underbrace{\mathrm{EE} = \|\mathrm{u} - \mathrm{u}_{\text{est}}\|_2^2}_{\text{End-point Error (in pixels)}}, \quad \text{and} \quad \underbrace{\mathrm{AE} = \cos^{-1}\left(\frac{1 + \mathrm{u}^{\mathrm{T}}\mathrm{u}_{\text{est}}}{\sqrt{1+\mathrm{u}^{\mathrm{T}}\mathrm{u}}\sqrt{1+\mathrm{u}_{\text{est}}^{\mathrm{T}}\mathrm{u}_{\text{est}}}}\right)}_{\text{Angular Error (in degrees)}}$$

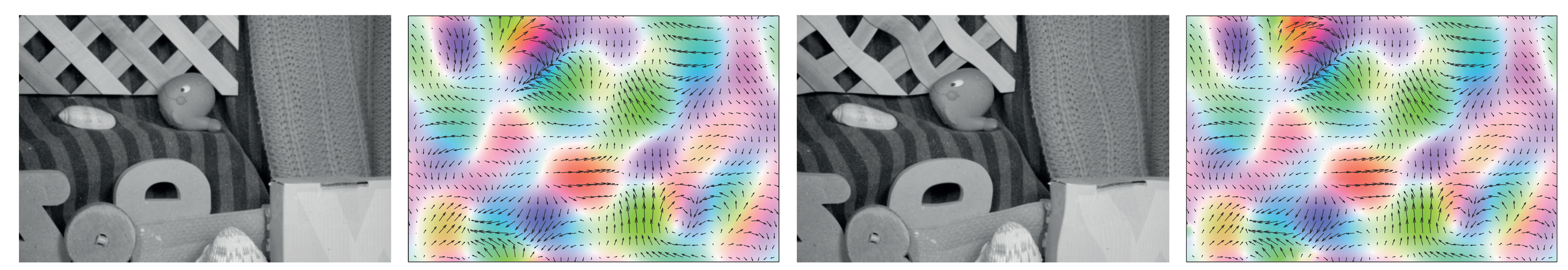
Comparison of the LAP algorithm against three state-of-the-art optical flow algorithms

	Constant Flows				Smoothly Varying Flows				Real Flows			
Algorithms	D = 1 pixel		D = 15 pixel		D = 1 pixel		D = 15 pixel		Dimetrodon		RubberWhale	
	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE	AAE	AEE
LAP	$\mathbf{4\times10^{-6}}$	$\mathbf{1\times10^{-7}}$	**0.001**	**0.001**	**0.107**	**0.002**	**0.746**	**0.102**	**1.782**	**0.096**	3.870	0.116
LDOF [3]	0.777	0.020	0.169	0.054	2.119	0.043	11.91	1.310	2.104	0.115	4.310	0.129
MPOF [4]	1.833	0.046	0.094	0.044	2.103	0.041	7.201	0.964	2.976	0.150	**2.662**	**0.087**
HS [1,6]	1.293	0.033	0.084	0.039	1.854	0.037	6.010	0.868	4.562	0.219	3.801	0.119

* AAE - Average Angular Error and AEE - Average End-point Error

** D is the maximum displacement of the optical flow

Estimating a smoothly varying optical flow with LAP algorithm (maximum displacement is 15 pixels)



(e) Image 1, I_1 (f) Ground Truth Flow, u (g) Image 2, I_2 (h) LAP Flow Estimate, u_{est}

Computation Time:

Computation time for the five optical flow algorithms (images are 388 by 584 pixels)

	LAP	LAP w. Median Filters	LDOF [3]	MPOF [4]	HS [1,6]
Time (seconds)	6.23	7.76	29.87	279.00	47.05

↬ Unlike the others, LAP computation times achieved using only a Matlab implementation

References

[1] B. Horn and B. Schunck, "Determining optical flow," *Artificial Intell.*, vol. 17, no. 1, pp. 185–203, 1981.

[2] B. Lucas and T. Kanade, "An iterative image registration technique with an application to stereo vision," in *Proc. Int. Joint Conf. Artificial Intell.*, Vancouver, Canada, 1981, vol. 2, pp. 674–679.

[3] T. Brox and J. Malik, "Large displacement optical flow: Descriptor matching in variational motion estimation," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 33, no. 3, pp. 500–513, 2011.

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[6] D. Sun, S. Roth, and M. Black, "A quantitative analysis of current practices in optical flow estimation and the principles behind them," *Int. J. Comput. Vision*, vol. 106, no. 2, pp. 115–137, 2014.