Covariance Cost Functions for Scheduling Multistatic Sonobuoy Fields

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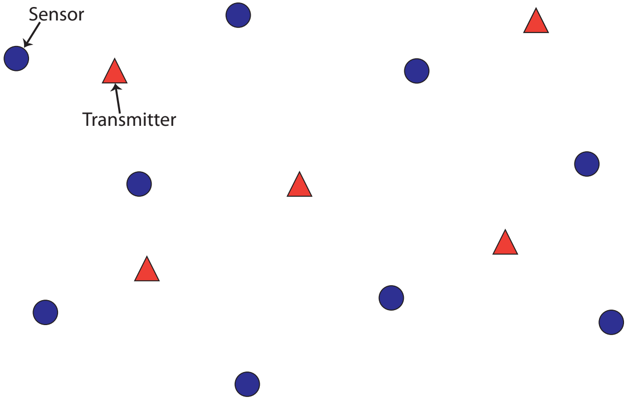
Outline

1. Multistatic Sonobuoy Fields
   - Two Tasks $\implies$ Search for and track underwater targets
   - Performance dependent on scheduling sonobuoys
2. Recap on Tracking in Sonobuoy Fields
   - Geometric Modelling and Measurements
   - Tracking algorithm used to track targets
3. Sonobuoy Scheduling Using Covariance-based Cost Functions
   - Tracking Reward Function
   - The 'size' of an ellipsoid
   - Covariance-based Cost Functions
4. Simulation Results
5. Conclusions

Multistatic Sonobuoy Fields

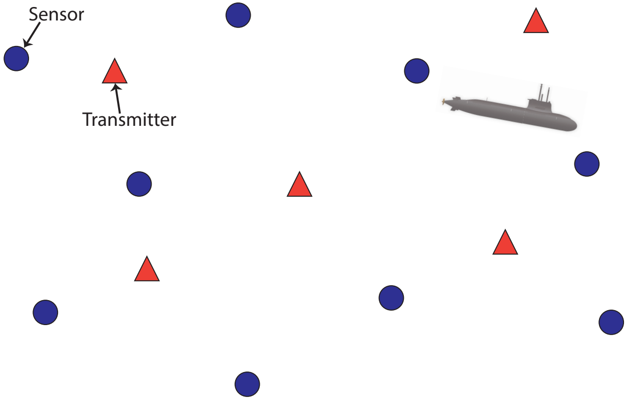
A network of transmitters and sensors distributed across a large search region



Multistatic Sonobuoy Fields

Two tasks of the system:

- Detect targets that are unknown to the system



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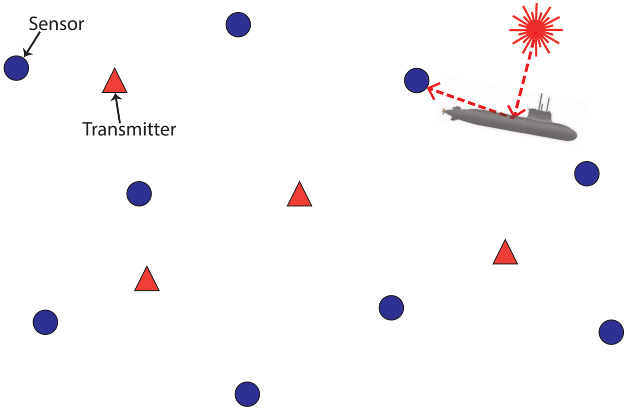

Multistatic Sonobuoy Fields

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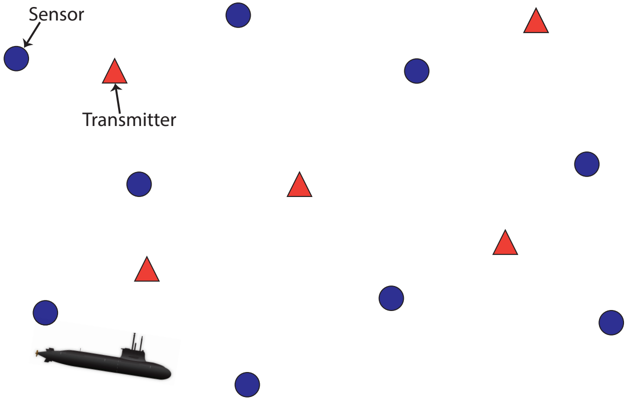
- Detect targets that are unknown to the system
- Accurately track targets known to the system



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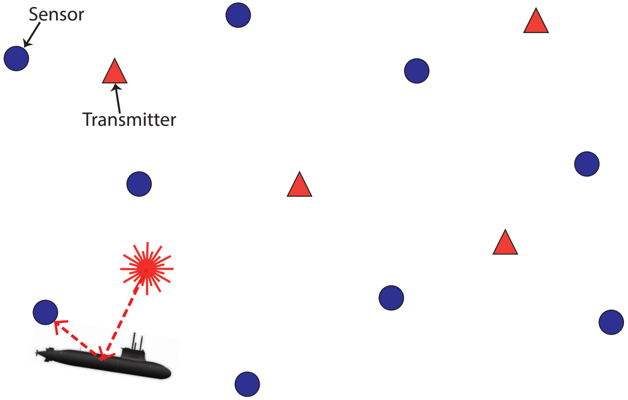

Scheduling Problem

- Choose sequence of transmitters and waveforms to satisfy tasks

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At one transmission time:

$$\text{Choose a Transmitter:} \quad \mathcal{T} = \{j_1, j_2, \ldots, j_{N_T}\}$$

where N_T is the number of transmitters in the field

$$\text{Choose a Waveform:} \quad \mathcal{W} = \{w_1, w_2, \ldots, w_{N_d}\}$$

where N_d is the number of possible waveforms

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Possible waveforms:

- Continuous Wave (CW) or Frequency Modulated (FM) waveform
- 1kHz or 2kHz frequency
- 2 second or 8 second duration

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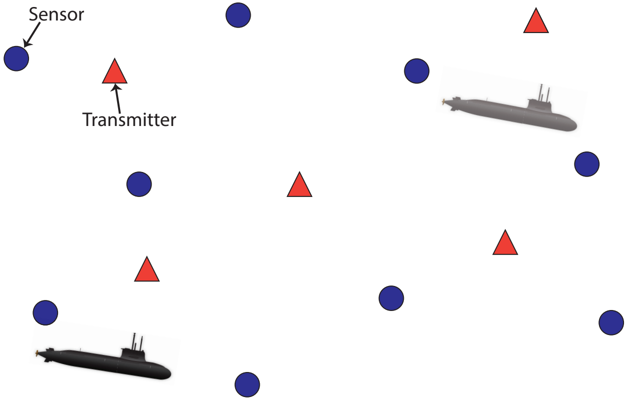
where N_d is the number of possible waveforms

Action space:

$$\text{Choose an action:} \quad a \in \mathcal{A}, \qquad \mathcal{A} = \mathcal{T} \times \mathcal{W}$$

Conflicting Objectives

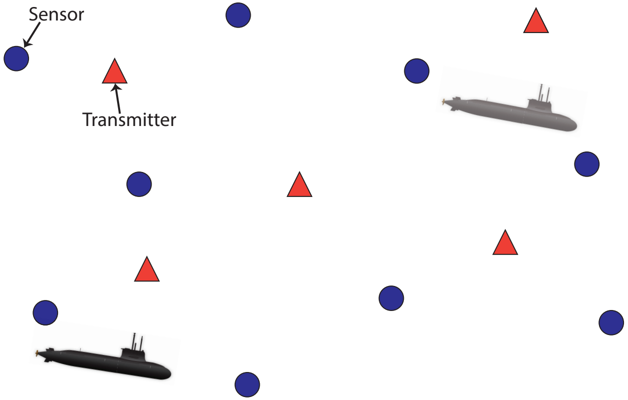
Track vs Search $\implies$ Which transmitter to choose...



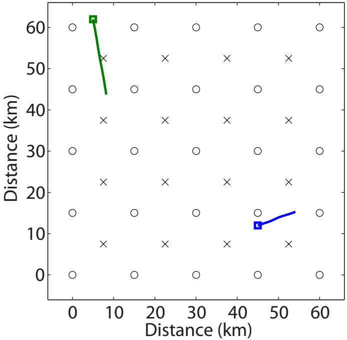
Conflicting Objectives

Our Approach:

Combine both tasks in multi-objective framework and use multi-objective optimization to decide scheduling



Modelling, Measurements & Tracking Algorithm



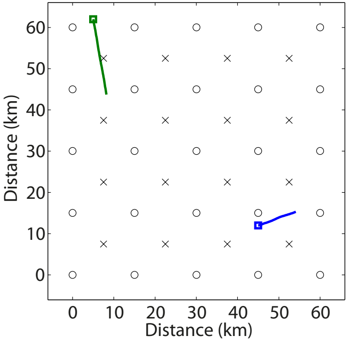
'x' = Transmitters, 'o' = Receivers

Sonobuoy Field Description:

- Transmitter positions
$$\mathbf{s}_j = \left[x_s^j, y_s^j\right]^{\mathrm{T}}$$
- Receiver positions
$$\mathbf{r}_i = \left[x_r^i, y_r^i\right]^{\mathrm{T}}$$
- Assume positions are known at all times*

*Each buoy contains RF communications and may contain GPS equipment

Modelling, Measurements & Tracking Algorithm

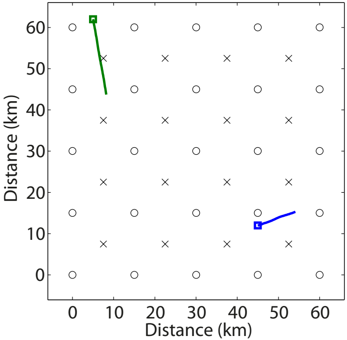


'x' = Transmitters, 'o' = Receivers

Target Description:

- Target Position at time t_k:
$$\mathbf{p} = [x_k, y_k]^{\mathrm{T}}$$
- Target Velocity at time t_k:
$$\mathbf{v} = [\dot{x}_k, \dot{y}_k]^{\mathrm{T}}$$
- Time-varying state
$$\mathbf{x}_k = [\mathbf{p}_k^{\mathrm{T}}, \mathbf{v}_k^{\mathrm{T}}]^{\mathrm{T}}$$

Modelling, Measurements & Tracking Algorithm



'x' = Transmitters, 'o' = Receivers

Target Motion:

- Noisy linear constant-velocity model

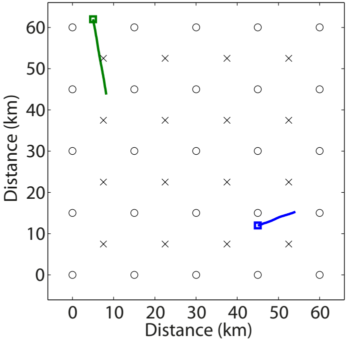
$$\mathbf{x}_k = \underbrace{\left(\begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \otimes \mathbf{I}_2 \right) \mathbf{x}_{k-1}}_{f(\mathbf{x}_{k-1})} + \mathbf{e}_k$$

- Process noise $\mathbf{e}_k$ is Gaussian with variance

$$\mathbf{Q} = \omega \begin{bmatrix} T^3/3 & T^2/2 \\ T^2/2 & T \end{bmatrix} \otimes \mathbf{I}_2$$

where $T = t_k - t_{k-1}$ is the sampling in time
$\otimes$ is the Kronecker product and $\mathbf{I}_2$ is 2×2 identity matrix

Modelling, Measurements & Tracking Algorithm

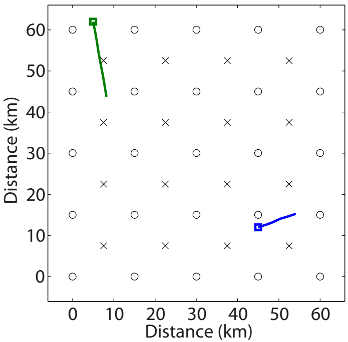


'x' = Transmitters, 'o' = Receivers

Measurements:

- Signal amplitude β and Kinematic measurement $\mathbf{z}$
$$\mathbf{z} = \mathbf{h}_j^{(i)}(\mathbf{x}_k) + \mathbf{w}_j^{(i)}$$
- Measurements collected from a subset of receivers
- Buoys have two waveform modalities
  - Frequency Modulated (FM)
  - Continuous Wave (CW)

Modelling, Measurements & Tracking Algorithm

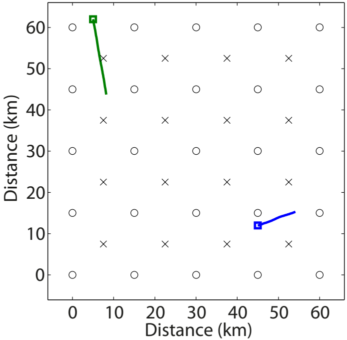


'x' = Transmitters, 'o' = Receivers

Using FM waveforms:

- Bistatic Range:
  $$|\mathbf{p}_k - \mathbf{r}_i| + |\mathbf{p}_k - \mathbf{s}_j|$$
- Angle from Receiver:
  $$\arctan\left(\frac{y_k - y_r^i}{x_k - x_r^i}\right)$$
- Good positional information

Modelling, Measurements & Tracking Algorithm

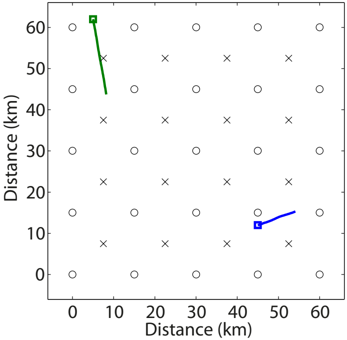


'x' = Transmitters, 'o' = Receivers

Using CW waveforms:

- Bistatic Range:
  $$|\mathbf{p}_k - \mathbf{r}_i| + |\mathbf{p}_k - \mathbf{s}_j|$$
- Angle from Receiver:
  $$\arctan\left(\frac{y_k - y_r^i}{x_k - x_r^i}\right)$$
- Bistatic Range-Rate:
  $$\mathbf{v}^{\mathrm{T}}\left[\frac{\mathbf{p}_k - \mathbf{r}_i}{|\mathbf{p}_k - \mathbf{r}_i|} + \frac{\mathbf{p}_k - \mathbf{s}_i}{|\mathbf{p}_k - \mathbf{s}_i|}\right]$$
- Good velocity information

Modelling, Measurements & Tracking Algorithm



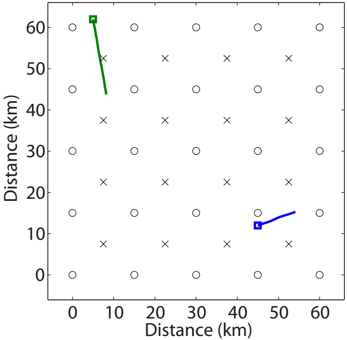
'x' = Transmitters, 'o' = Receivers

Tracking Challenges:

- High levels of clutter
- Non-linear measurements
- Low probability of detection

Many possible algorithms: ML-PDA, MHT, PMHT, JIPDA, PHD/CPHD, ... etc

Modelling, Measurements & Tracking Algorithm



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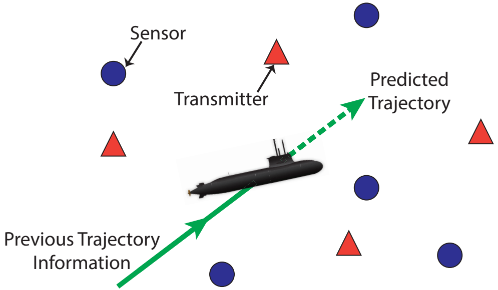
The tracker:

- Multi-Sensor Bernoulli filter[1] (optimal multi-sensor Bayesian filter for a single target)
- Linear Multi-Target (LMT) Paradigm[2]
- Gaussian mixture model implementation[3]
- Process FM & CW measurements

[1] B. Ristic *el al.*, '*A tutorial on Bernoulli filters: Theory, implementation and applications*', IEEE Trans. Signal Process., 2013.
[2] D. Mušicki and B. La Scala, '*Multi-Target Tracking in Clutter without Measurement Assignment*', IEEE Trans. Aerosp. Electron. Syst., 2008.
[3] B. Ristic *el al.*, '*Gaussian Mixture Multitarget Multisensor Bernoulli Tracker for Multistatic Sonobuoy Fields*', IET Radar, Sonar & Navig., 2017.

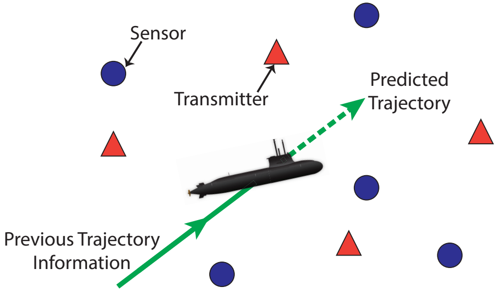
Tracking Reward



Given previous tracking:

↬ Measure the gain in tracking information from action a

Tracking Reward

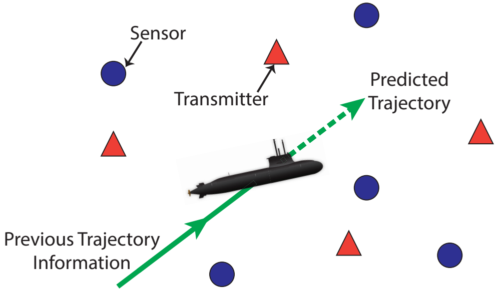


Approximate information matrix:

Single track: $$\text{trace}\left[\mathbf{J}_{\text{Predict}} + \sum_{i \in \mathcal{R}} P_d^i(a)\mathbf{J}^i_{\text{Measure}}(a)\right]$$

Trace of only the positional elements of information matrix
$P_d^i(a)$ Expected probability of detecting track

Tracking Reward

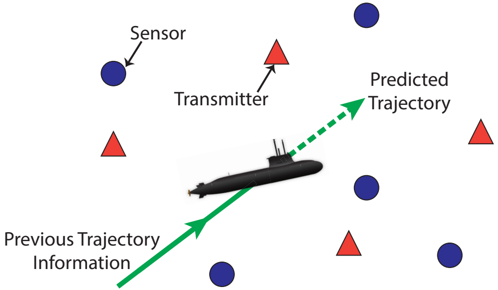


Predicted Information Matrix:

$$\underbrace{\mathbf{J}_{\text{Predict}} = \left[\mathbf{F}_{k-1}\mathbf{P}_{k-1}\left[\mathbf{F}_{k-1}\right]^{\mathrm{T}}\right]^{-1}}_{\text{Propagation of error covariance due to motion model}}$$

where $\mathbf{F}_{k-1}$ is the Jacobian of $f(\mathbf{x}_{k-1})$ and $\mathbf{P}_{k-1}$ is the error covariance from tracker

Tracking Reward



Measurement Information Matrix:

$$\underbrace{\mathbf{J}_{\text{Measure}} = \left[\mathbf{H}_k^i(a)\right]^{\mathrm{T}} \left[\mathbf{R}_k^i(a)\right]^{-1} \mathbf{H}_k^i(a)}_{\text{Gain in information from action}}$$

where $\mathbf{H}_k^i(a)$ is the Jacobian of $h_a(\mathbf{x}_{k-1})$ and $\mathbf{R}_k^i(a)$ is the measurement covariance

How big is an ellipse

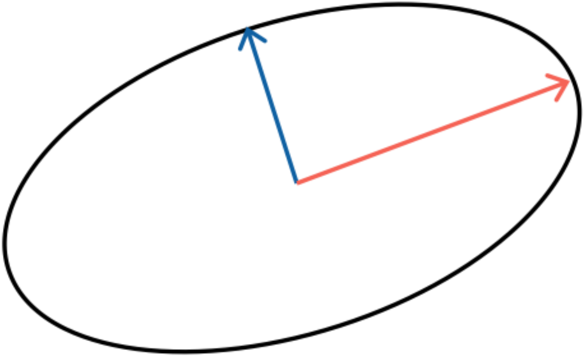


Figure: Ellipse showing the eigenvectors of the matrix definition of an ellipse, $x\Sigma x^T = C$

Possible Cost Functions

Analysed scheduling of multistatic sonobuoy fields using cost functions based on the eigenvalues of the covariance estimate, $\mathbf{P}(a)$, $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_n$

1. $\mathcal{J}_{\text{trace}}(a) = \operatorname{trace} \mathbf{P}(a) = \sum_n \lambda_n$
2. $\mathcal{J}_{\text{det}}(a) = \det \mathbf{P}(a) = \prod_n \lambda_n$
3. $\mathcal{J}_{\text{pres}}(a) = 1 / \operatorname{trace} \mathbf{P}(a)^{-1}$
4. $\mathcal{J}_{\text{max}}(a) = \max_n \lambda_n = \lambda_1$
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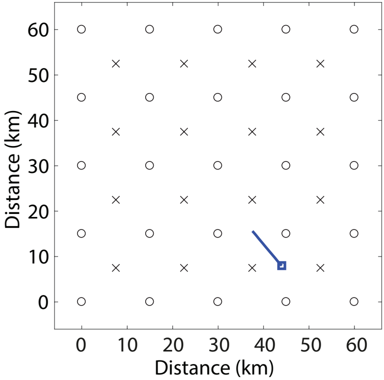
Analysis of Scheduler - Set Up



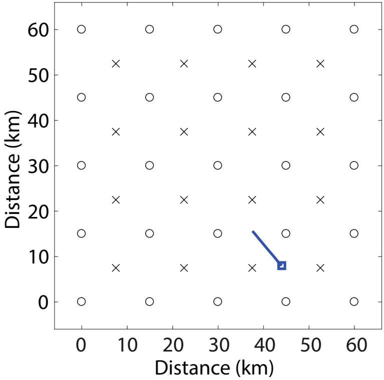
'x' = Transmitters, 'o' = Receivers

Set-up:

- 4 × 4 transmitter grid
- 5 × 5 receiver grid
- Buoy separation = 15km
- 50 Minute Scenario
- 1 transmission/minute
- 600 Simulations

Realistic measurements $\implies$ Bistatic Range Independent Signal Excess (BRISE) simulation environment

Analysis of Scheduler - Set Up



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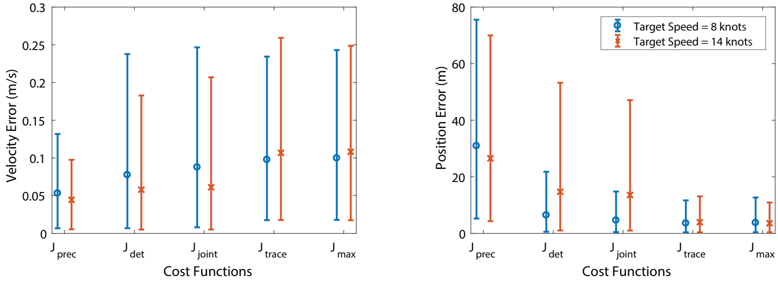
Analyse the performance of the scheduler as cost function varies

Analysis of Scheduler - Results

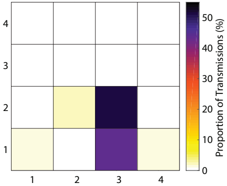


(a) Velocity Error (m/s)

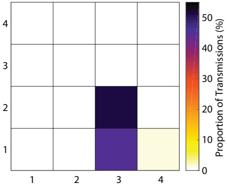
(b) Position Error (m)

Figure: Performance of the scheduler shows the mean position and velocity error obtained using the different cost functions. The blue lines for target speed of 8 knots, and the red lines for a target speed of 14 knots

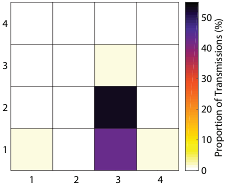
Analysis of Scheduler - Transmitter Choice



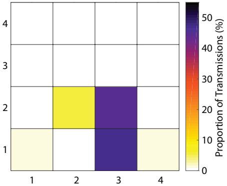
(a) Using $\mathcal{J}_{\text{prec}}$



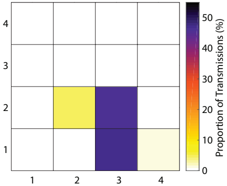
(b) Using $\mathcal{J}_{\text{det}}$



(c) Using $\mathcal{J}_{\text{joint}}$



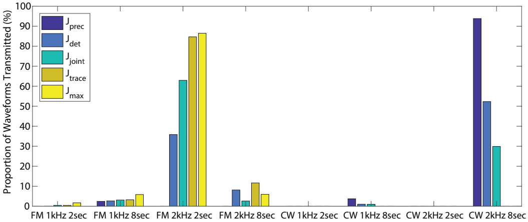
(d) Using $\mathcal{J}_{\text{trace}}$



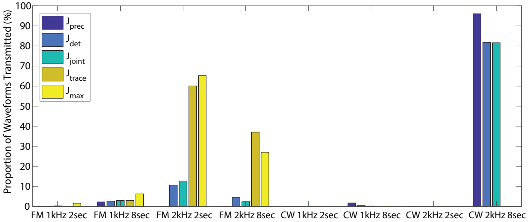
(e) Using $\mathcal{J}_{\max}$

Figure: Transmitter histograms for the target's speed of 14 knots, showing the proportion of transmissions from each source when using the different cost functions.

Analysis of Scheduler - Waveform Choice



(a) Target Speed $= 8\,$knots



(b) Target Speed $= 14\,$knots

Figure: Choice of waveforms used when the target's speed is 8 and 14 knots.

Analysis of Scheduler - Results

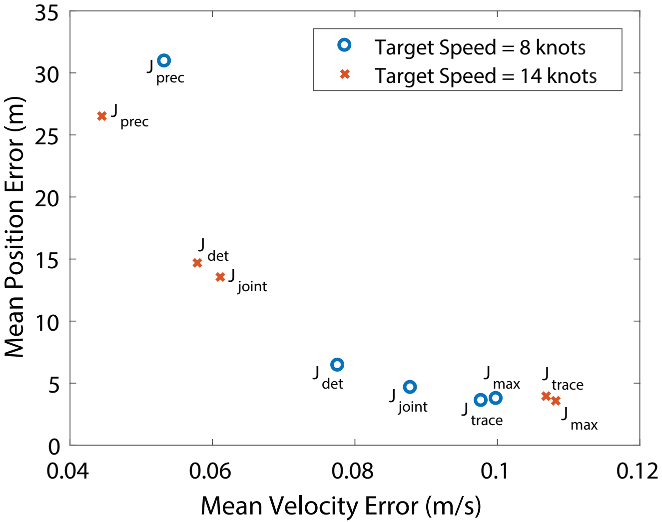


Figure: the means of both errors on a scatter

Conclusions

- Analysed scheduling of multistatic sonobuoy fields using cost functions based on the eigenvalues of the covariance estimate, $\mathbf{P}(a)$, $\lambda_1 \geq \lambda_2 \ldots \geq \lambda_n$
  1. $\mathcal{J}_{\text{trace}}(a) = \operatorname{trace} \mathbf{P}(a) = \sum_n \lambda_n$
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  5. $\mathcal{J}_{\text{joint}}(a) = \lambda^x \lambda^v$
- Analysed proposed scheduling via optimum source-waveform action that minimized the covariance cost function.
  - Demonstrated trade-off between position and velocity accuracy varies
  - Trade-off characterised in terms of points on the Pareto front

The End

Thank you for listening
Any Questions?